25. $f(x) = e^{\frac{x}{2}}$ $\quad f(0) = 1$

$e^{x} = 1 + x + \frac{x^2}{2} \ldots$

$e^{\frac{x}{2}} = 1 + \frac{1}{2}x + \frac{1}{4}\cdot\frac{x^2}{2}$

a) $f'(x) = \frac{1}{2}e^{x/2}$ $\quad f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{4}e^{x/2}$ $\quad f''(0) = \frac{1}{4}$

b) $g(x) = \frac{e^{x}-1}{x}$ $\quad e^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \ldots$

$\frac{e^{x}-1}{x} = \frac{x}{x} + \frac{x^2}{x \cdot 2} + \frac{x^3}{x \cdot 3!} \ldots$

c) $g'(x) = \frac{x e^{x} - (e^{x}-1)\cdot 1}{x^2}$

$g(x) = 1 + \frac{x}{2} + \frac{x^2}{3!} + \frac{x^3}{4!} \ldots$

$g'(1) = \frac{1\cdot e - e + 1}{1} = 1$

$g(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}$

$g'(1) = \sum_{n=0}^{\infty} \frac{n}{(n+1)!} = 1$

$g'(x) = \sum_{n=0}^{\infty} \frac{n x^{n-1}}{(n+1)!}$

$g'(x) = \frac{1}{2} + \frac{2x}{3!} + \frac{3x^2}{4!} \ldots$

$g'(1) = \frac{1}{2} + \frac{2}{3!} + \frac{3}{4!} \ldots = 1$

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24. $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} \ldots + \frac{x^n}{(n+1)!} \ldots$

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 \ldots \frac{f^{(n)}(0)}{n!}x^n \ldots$

$f'(0)x = \frac{x}{2!}$

$f'(0) = \frac{1}{2}$

$f^{(10)}(0) = \frac{10!}{11!} = \frac{1}{11}$ $\qquad \frac{f^{(10)}(0)}{10!}x^{10} = \frac{x^{10}}{11!}$

$\frac{\cancel{10}\cdot\cancel{9}\cdot\cancel{8}\cdots\cancel{1}}{11\cdot\cancel{10}\cdot\cancel{9}\cdots\cancel{1}}$ $\qquad \frac{f^{(10)}(0)}{10!} = \frac{1}{11!}$

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9.2b Taylor Series

Maclaurin series you should have memorized: p 491

$$\boxed{\begin{aligned}
e^{x} &= 1 + x + \frac{x^2}{2} \ldots + \frac{x^n}{n!} \ldots && n \text{ starts at } 0\\
\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots + \frac{x^{2n+1}}{(2n+1)!}(-1)^n && n \text{ starts at } 0\\
\cos x &= 1 - \frac{x^2}{2} + \frac{x^4}{4!} \ldots + \frac{x^{2n}}{(2n)!}(-1)^n && n \text{ starts at } 0\\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} \ldots + \frac{x^n}{n}(-1)^{n+1} && n \text{ starts at } 1
\end{aligned}}$$

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Taylor series for f(x) centered on x=a

$$\boxed{f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 \ldots \frac{f^{(n)}(a)}{n!}(x-a)^n \ldots}$$

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Find the Taylor series generated by f(x)=e^x at x=2

$$e^x = e^2 + e^2(x-2) + \frac{e^2}{2}(x-2)^2 + \frac{e^2}{3!}(x-2)^3 \cdots$$

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Find the Maclaurin series for $f(x) = \frac{1+\cos(2x)}{2}$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} \cdots$$

$$\cos(2x) = 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{4!} \cdots$$

$$\frac{1+\cos(2x)}{2} = \frac{2}{2} - \frac{(2x)^2}{2\cdot 2} + \frac{(2x)^4}{2\cdot 4!} \cdots$$

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Find the first four nonzero terms for the Maclaurin series for $\sin(x^2)$
Use this series to find a corresponding series for $\int \sin(x^2)\,dx$
(why not just evaluate $\int \sin(x^2)\,dx$?)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots$$

$$\sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} \cdots$$

$$\int \sin(x^2) = \int x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} \cdots$$

$$\int \sin(x^2)dx = \frac{x^3}{3} - \frac{x^7}{3!\,7} + \frac{x^{11}}{5!\,11} \cdots$$

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